

# Calibration of rectangular atomic force microscope cantilevers

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A method to determine the spring constant of a rectangular atomic force microscope cantilever is proposed that relies solely on the measurement of the resonant frequency and quality factor of the cantilever in fluid (typically air), and knowledge of its plan view dimensions. This method gives very good accuracy and improves upon the previous formulation by Sader *et al.* [Rev. Sci. Instrum. **66**, 3789 (1995)] which, unlike the present method, requires knowledge of both the cantilever density and thickness. © 1999 American Institute of Physics. [S0034-6748(99)04210-0]

## I. INTRODUCTION

The ability to experimentally determine the spring constants of atomic force microscope (AFM) cantilevers is of fundamental importance in many applications of the AFM. To date many methods have been devised for this purpose. These include methods that monitor the static deflection of the cantilevers<sup>1,2</sup> and those that utilize the cantilevers' dynamic deflection properties.<sup>3-5</sup> Recently, Sader *et al.*<sup>5</sup> proposed a method whereby the spring constant of an AFM cantilever is determined from its unloaded resonant frequency. The mass of the cantilever is also required and this is typically obtained from the density, thickness, and plan view dimensions of the cantilever. In particular, for the case of a rectangular cantilever, the spring constant is given by

$$k = M_e \rho_c b h L \omega_{\text{vac}}^2, \quad (1)$$

where  $\omega_{\text{vac}}$  is the fundamental radial resonant frequency of the cantilever in vacuum,  $h$ ,  $b$ , and  $L$  are the thickness, width, and length of the cantilever, respectively,  $\rho_c$  is the density of the cantilever, and  $M_e$  is the normalized effective mass which takes the value  $M_e = 0.2427$  for  $L/b > 5$ .<sup>5</sup> Although simple in appearance, application of Eq. (1) has been limited for several practical reasons that we shall now discuss. In contrast to the plan view dimensions of the cantilever that are easily measured using optical techniques, the thickness measurement typically requires the use of electron microscopy which can be time consuming and cannot be routinely carried out on every cantilever. Furthermore, determination of the density or mass of the cantilever can pose an even greater difficulty. Typically, the reflectivity of AFM cantilevers is increased by the deposition of thin gold films onto the substrate Si or Si<sub>3</sub>N<sub>4</sub>. This also necessitates the deposition of a thin sputter-coated chromium layer to improve adhesion of the gold layer. For most commercial cantilevers the thickness of these individual layers is unknown, so that the average density of the cantilevers, and hence their mass, remain completely undetermined.<sup>5</sup> Also, measurements of the frequency response of AFM cantilevers are commonly performed in an

air or liquid medium. It is well known that the surrounding medium can significantly reduce the resonant frequency from its value in vacuum.<sup>5</sup> Hence, if not accounted for, this effect can introduce an unnecessary and significant error into the calibration procedure.<sup>5</sup>

In an effort to understand the role of the surrounding fluid medium on cantilever dynamics, we recently examined the effect of immersing a rectangular AFM cantilever in a viscous fluid.<sup>6,7</sup> There it was found that the shift in resonant frequency of the cantilever from vacuum to fluid (gas or liquid) was strongly dependent on both the density and viscosity of the fluid and it could be accurately modeled. We now show that these same results can be used to formulate a simple, practical, and accurate method for the determination of the spring constants of rectangular AFM cantilevers. The present method of calibration eliminates the requirement for *a priori* knowledge of the density and thickness, and automatically accounts for the effects of the surrounding fluid, thus overcoming the problems mentioned above. The spring constant of a rectangular cantilever is then determined from its plan view dimensions, and the measurement of its resonant frequency and quality factor in fluid (typically air). All these quantities are readily accessible and facilitate fast and accurate determination of the spring constant by the present method. The following formulation pertains to a rectangular cantilever whose length  $L$  greatly exceeds its width  $b$  which in turn greatly exceeds its thickness  $h$ , which is the case commonly encountered in practice. We shall examine the practical limitations of this assumption and assess the accuracy and validity of the present method by giving a detailed comparison with experimental measurements.

## II. THEORY

To begin we note that the shift in the resonant frequency from vacuum to fluid is primarily due to inertial effects in the fluid.<sup>6,7</sup> It then follows that if the quality factor  $Q_f$  of the fundamental mode of the cantilever in fluid greatly exceeds 1 (which is typically satisfied when the cantilever is placed in air<sup>6-8</sup>), the vacuum resonant frequency  $\omega_{\text{vac}}$  is related to the resonant frequency in fluid  $\omega_f$  by<sup>6</sup>

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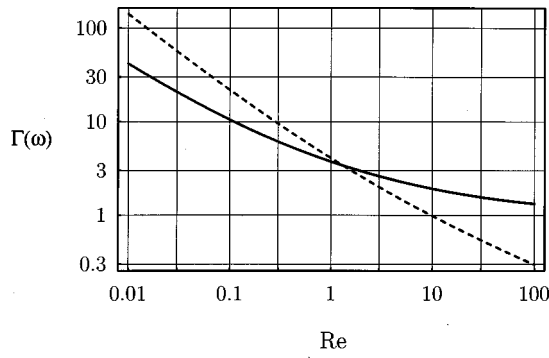


FIG. 1. Plot of the real and imaginary components of the hydrodynamic function  $\Gamma(\omega)$  as a function of the Reynolds number  $\text{Re} = \rho_f \omega b^2 / (4\eta)$ . The real component  $\Gamma_r$  is shown by the solid line; the imaginary component  $\Gamma_i$  is shown by the dashed line.

$$\omega_{\text{vac}} = \omega_f \left( 1 + \frac{\pi \rho_f b}{4 \rho_c h} \Gamma_r(\omega_f) \right)^{1/2}, \quad (2)$$

whereas the areal mass density  $\rho_c h$  is given by<sup>6</sup>

$$\rho_c h = \frac{\pi \rho_f b}{4} [Q_f \Gamma_i(\omega_f) - \Gamma_r(\omega_f)], \quad (3)$$

where  $\rho_f$  is the density of the fluid, and  $\Gamma_r$  and  $\Gamma_i$  are, respectively, the real and imaginary components of the hydrodynamic function  $\Gamma$ , which is plotted in Fig. 1. The reader is referred to Eq. (20) of Ref. 6 for an analytical expression for  $\Gamma$ . We note that  $\Gamma(\omega)$  only depends on the Reynolds number  $\text{Re} = \rho_f \omega b^2 / (4\eta)$ , where  $\eta$  is the viscosity of the surrounding fluid, and is independent of the cantilever thickness and density. Substituting Eqs. (2) and (3) into Eq. (1) we obtain the required result

$$k = 0.1906 \rho_f b^2 L Q_f \Gamma_i(\omega_f) \omega_f^2. \quad (4)$$

Equation (4) relates the spring constant  $k$  directly to the plan view dimensions of the cantilever, the fundamental mode resonant frequency  $\omega_f$ , and the quality factor  $Q_f$  in fluid. This expression is valid provided the quality factor  $Q_f \gg 1$ , which is typically satisfied when the cantilevers are placed in air. We also note that if the areal mass density and vacuum frequency of the cantilever are required, then these can be easily obtained from Eqs. (2) and (3).

### III. EXPERIMENTAL RESULTS

We now assess the accuracy of the present method by presenting a detailed comparison with experimental measurements. Two different sets of cantilevers were chosen for this comparison. The first set, henceforth referred to as *calibrated cantilevers*, were procured from Park Scientific Instruments<sup>9</sup> and have a highly uniform rectangular geometry, with accurately controlled material properties and dimensions.<sup>10</sup> Consequently, the spring constants of these cantilevers are specified to a high tolerance by the manufacturer.<sup>9,10</sup> The second set of cantilevers were obtained from Digital Instruments,<sup>11</sup> and are identical to the cantilevers used by Walters *et al.*<sup>8</sup> These latter cantilevers are gold coated and have a nonideal geometry (i.e., the end is cleaved

TABLE I. Comparison of spring constants for calibrated cantilevers (Ref. 10) determined by the manufacturer  $k_{\text{man}}$  and by the present method  $k_{\text{new}}$ .  $f_f$  and  $Q_f$  are the resonant frequency and quality factor in air, respectively. The results for 397 and 197  $\mu\text{m}$  cantilevers were obtained from their thermal noise spectra, whereas the results for the 97  $\mu\text{m}$  cantilever were obtained by driving the cantilever. Dimensions of the cantilevers:  $b = 29 \mu\text{m}$  and  $h = 2 \mu\text{m}$ . Properties of air:  $\rho_f = 1.18 \text{ kg m}^{-3}$  and  $\eta = 1.86 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$ .

$L$ ( $\mu\text{m}$ )	$f_f$ (kHz)	$Q_f$	$k_{\text{man}}$ ( $\text{N m}^{-1}$ )	$k_{\text{new}}$ ( $\text{N m}^{-1}$ )
397	17.36	55.5	0.157	0.157
197	69.87	136	1.3	1.30
97	278.7	309	10.4	10.7

and a tip is added; see Ref. 8 for details). The spring constants of these cantilevers were measured using the method of Cleveland *et al.*<sup>4</sup> These results served as the benchmark for comparison with the present method. All measurements made using the present method were performed in air, thus satisfying the criterion that  $Q_f \gg 1$ . To obtain the quality factor  $Q_f$  and resonant frequency  $\omega_f$ , as is required in the present method, we measured the thermal noise spectra of the cantilevers using a National Instruments data acquisition (DAQ) card,<sup>12</sup> carrying out a digital fast Fourier transform of the signal using LabVIEW software.<sup>13</sup> The fundamental mode resonance peaks of the measured spectra were then fitted to the response of a simple harmonic oscillator,<sup>6,14</sup> using a nonlinear least squares fitting procedure.<sup>15</sup> A white noise floor was included in the fitting procedure<sup>14</sup> to ensure accurate fits to the noise spectra. In some cases, a large spring constant precluded the measurement of the thermal noise spectrum, due to low cantilever vibration amplitude. For these situations, the frequency response of the cantilever was measured by driving the cantilever at a range of frequencies in the neighborhood of its fundamental mode resonance peak.<sup>16</sup> We emphasize that this approach gives the same results for  $Q_f$  and  $\omega_f$  as that obtained from the thermal noise spectrum, provided the drive amplitude is not large enough to invoke nonlinearities. These two approaches allowed easy and accurate determination of the quality factor ( $\pm 1\%$ ) and resonant frequency ( $\pm 0.1\%$ ). The plan view dimensions of the cantilevers were measured using an optical microscope.

In Table I, we present results for the calibrated cantilevers.<sup>10</sup> For each cantilever, the method by which its resonant frequency and quality factor was obtained is indicated. Note the very good accuracy achieved by the present method for all cantilevers, whose aspect ratios range from  $L/b = 3.3 - 13.7$ . In Table II, we present results for the second set of cantilevers which are gold coated and have a nonideal geometry.<sup>8</sup> Again note the good accuracy obtained by the present method in comparison to the method of Cleveland *et al.*<sup>4</sup> for all cantilevers, which range in aspect ratio from  $L/b = 3.9 - 10$ . For these cantilevers, the thermal noise spectrum was used to evaluate  $Q_f$  and  $\omega_f$  in all cases. From these results it is clear that the present method is capable of accurate determination of the spring constants of rectangular AFM cantilevers, whose aspect ratios  $L/b$  exceed 3,<sup>17</sup> which is the case commonly encountered in practice. The present method overcomes the shortcomings of the method proposed in Ref. 5 by eliminating the requirement for knowledge of

TABLE II. Comparison of spring constants for gold coated cantilevers (Ref. 8) determined by the method of Cleveland *et al.* (Ref. 4)  $k_{\text{clv}}$  and by the present method  $k_{\text{new}}$ .  $f_f$  and  $Q_f$  are the resonant frequency and quality factor in air, respectively. Dimensions of the cantilevers:  $b=20 \mu\text{m}$  and  $h=0.44 \mu\text{m}$ . Properties of air:  $\rho_f=1.18 \text{ kg m}^{-3}$  and  $\eta=1.86 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$ .

$L$ ( $\mu\text{m}$ )	$f_f$ (kHz)	$Q_f$	$k_{\text{clv}}$ ( $\text{N m}^{-1}$ )	$k_{\text{new}}$ ( $\text{N m}^{-1}$ )
203	10.31	17.6	0.010	0.010
160	15.61	22.7	0.019	0.018
128	24.03	30.9	0.038	0.034
105	36.85	41.7	0.064	0.067
77	64.26	60.3	0.15	0.15

the cantilever's thickness, density, and resonant frequency in vacuum, parameters that in practice can be difficult to measure.

#### IV. NON-RECTANGULAR CANTILEVERS

Finally, we note that the present method can also be used indirectly to calibrate cantilevers of different geometries, as we shall now discuss. If all cantilever chips manufactured had a single rectangular cantilever attached, as is the case in some chips currently produced,<sup>9</sup> then the calibration of all cantilevers could be made very simple and routine using the present method. This would utilize the property that the thickness and material properties of all cantilevers over a single chip are typically constant. The procedure would then involve calibration of the rectangular cantilever, from which the product  $Eh^3$ , where  $E$  is Young's modulus, could be easily evaluated using

$$Eh^3 = k \frac{4L^3}{b}. \quad (5)$$

Subsequently, theoretical results for the spring constants, such as those found in Refs. 18–21, could then be used to calibrate cantilevers of other geometries. This would require only the plan view dimensions of the cantilevers, which are easily measured.

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<sup>11</sup>Digital Instruments, 112 Robin Hill Road, Santa Barbara, CA 93117.

<sup>12</sup>AT-MIO-16E-1 board, available from National Instruments, 6504 Bridge Point Parkway, Austin, TX 78730-5039.

<sup>13</sup>LabVIEW is a registered trademark of, and is available from, National Instruments (see Ref. 12).

<sup>14</sup>The resonant frequency  $\omega_f$  and quality factor  $Q_f$  were obtained by fitting the power spectra to the functional  $P(\omega) = A_{\text{white}} + B\omega_f^4 / [(\omega^2 - \omega_f^2)^2 + \omega^2 \omega_f^2 / Q_f^2]$ . The fitting parameters are  $A_{\text{white}}$ ,  $B$ ,  $\omega_f$ , and  $Q_f$ .

<sup>15</sup>Mathematica is a registered trademark of, and is available from, Wolfram Research, Inc., 100 Trade Center Drive, Champaign, IL 61820-7237.

<sup>16</sup>The cantilevers were vibrating at their base using the Tapping Mode cantilever tune software in the Nanoscope III atomic force microscope (Digital Instruments, USA).

<sup>17</sup>To establish the ultimate lower limit for  $L/b$ , for which the method is applicable, measurements need to be performed on cantilevers with aspect ratios smaller than those used in this study.

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